A homoclinic route to asymptotic full cooperation in adaptive networks and its failure

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Abstract. We consider the evolutionary dynamics of a cooperative game on an adaptive network, where the strategies of agents (cooperation or defection) feed back on their local interaction topology. While mutual cooperation is the social optimum, unilateral defection yields a higher payoff and undermines the evolution of cooperation. Although no *a priori* advantage is given to cooperators, an intrinsic dynamical mechanism can lead asymptotically to a state of full cooperation. In finite systems, this state is characterized by long periods of strong cooperation interrupted by sudden episodes of predominant defection, suggesting a possible mechanism for the systemic failure of cooperation in real-world systems.

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1. Introduction

Understanding the evolution of cooperation between selfish players is of importance not only because of its central role in biology and the social sciences [1, 2] but also to address the emergence and failure of cooperation in human societies [3]. Previous work has identified several key mechanisms [4] leading to the evolution and fixation of cooperation. Among others, it was found that spatial structure can promote cooperation [5] when agents are placed on regular lattices [6] or on complex networks [7–9]. Perhaps the newest development in this direction is the investigation of games on adaptive networks [10], in which the players' behaviour feeds back on the network topology. Cooperation is promoted if cooperating players can secure an advantageous topological position, directly due to avoidance of defectors [11–17], or indirectly due to the continuous arrival of new players [18] or other ongoing changes of the topology [19, 20]. Furthermore, cooperation in adaptive networks can profit from the emergence of a self-organized leadership structure [21–24] and the formation of strongly heterogeneous topologies [25–27]. Finally, it was shown that cooperation on adaptive networks can result in an arms race for swift topological response that promotes cooperation [28, 29]. Interestingly, full cooperation of every single agent has been observed in a number of studies. This is explained in part in [30], showing that in the limit of fast rewiring, cooperative games on adaptive networks can be mapped on coordination games in a well-mixed population. The coevolutionary games listed here and other forms thereof have been reviewed in [31].

Although cooperation on adaptive networks is an inherently dynamical process, previous studies in this field have focused mainly on the average, and therefore quasi-static, level of cooperation that evolves. Dynamical phenomena such as the appearance of oscillations are known to play a role in non-adaptive models of iterated and cyclic games, where they have been investigated thoroughly (e.g., [32–34]). Furthermore, interesting dynamical behaviour has been observed already in a pioneering publication on adaptive networks [21] and many subsequent works [13, 17, 19, 21–24] and has been analysed to some extent in [35]. Only recently, oscillations have been observed in an adaptive network model, where they were interpreted in terms of a Red Queen mechanism that is also able to promote cooperation [20].

In the present paper, we study an evolutionary game on an adaptive network where the collective dynamics of the agents lead to asymptotic full cooperation. In contrast to previous work that mainly focused on local update rules based on the payoff of neighbouring nodes [11, 21–23], the agents use non-local information about the general performance of the strategies in our model. This corresponds to the accessibility of certain global knowledge in social systems through, e.g., the media.

Investigating the dynamics of the present model, we find oscillations in the number of cooperating players, in which the time-averaged payoff of the latter equals that of the defectors in the limit of infinite population size. Nevertheless, a state of full cooperation is approached asymptotically if the rate of topological change exceeds a finite threshold. This is made possible by a dynamical mechanism involving the formation of a homoclinic loop in a global bifurcation. Combining ideas from network science, nonlinear dynamics, and evolutionary game theory, we derive a low-dimensional analytical model explaining this dynamic phenomenon far from equilibrium. Furthermore we show that in finite populations, this mechanisms can lead to periods of almost full cooperation interrupted by recurrent collapses to episodes of predominant defection, revealing a scenario for the sudden failure of cooperation.

2. Model

We consider an undirected network of N nodes, representing agents, and K links, representing interactions. Each agent i is assigned a strategy σ_i, which can either be cooperation, $C := 1$, or defection, $D := 2$. The payoff gained in an interaction is modelled by the snowdrift game, a paradigmatic model in game theory [36]. In a common parametrization, two interacting agents receive a benefit b if either of them cooperates. Cooperation incurs a cost $c < b$, which is divided between cooperators, but not defectors. The payoff a player i receives from the interaction with player j can then be written as $M_{\sigma_i\sigma_j}$, where

$$\mathbf{M} = \left(\begin{array}{cc} b - \frac{c}{2} & b - c \\ b & 0 \end{array} \right) \tag{1}$$

is the payoff matrix. The total payoff player i gains from all interactions is given by $\pi_i = \sum_{j \sim i} M_{\sigma_i\sigma_j}$, where the summation runs over all j linked to i.

Starting from a random graph and randomly assigned equiprobable strategies, we evolve the network as follows: In every time step, one link is selected at random. With probability p, this focal link is rewired. Otherwise, i.e., with probability $q = 1 - p$, one of the linked players adopts the other player's strategy. For large p, players thus tend to change their interaction partners, whereas for small p they tend to revise their behaviour.

To complete the model, we have to specify which agent copies the other's strategy in a strategy adoption event and which agent keeps the link when rewiring takes place. Most previous models assume that the agents' access to information is governed by the same network as the underlying games, forcing the agents to base their decisions on information from direct neighbours. The same network topology thus determines three different aspects of the system: the interaction partners of an agent, against whom the game is played, the potential role models, whose strategies can be adopted, and the agents from which information can be obtained. However, for intelligent agents, and especially humans, there is no reason to assume that these three network roles are all fulfilled by coinciding topologies [37]. In fact, the use of identical networks for information and state transmission has recently been criticized for the related application of epidemic spreading [38, 39].

In the present paper, we assume that information transfer in the population is not governed exclusively by the interaction network. As a first approximation, we consider the simplest case in which the information transmission network is replaced by an effective global coupling, as information can be rapidly transmitted and in the human population is also transported by the mass media. In the main part of this work, we assume that the agents rely on their perception of a strategy's *general* performance. This global measure for a strategy σ is obtained as the average payoff of all agents currently using σ,

$$\phi(\sigma) = \sum_{i, \sigma_i = \sigma} \frac{\pi_i}{n_\sigma N} = \sum_{\sigma' \in \{C, D\}} (1 + \delta_{\sigma\sigma'}) M_{\sigma\sigma'} \frac{l_{\sigma\sigma'}}{n_\sigma}, \tag{2}$$

where δ is the Kronecker delta, n_σ is the fraction of the population using the strategy, and $l_{\sigma\sigma'}$ is the number of links between agents using strategies σ and σ' normalized by N. The validity of assuming such a measure based on global information is discussed in section 4, where we consider the effect of a finite "information horizon" within which the agents assess the performance of a strategy. If strategy adoption occurs on a link connecting the agents i and j, then we assign the strategy of agent i to agent j with probability given by the Fermi function [40, 41]

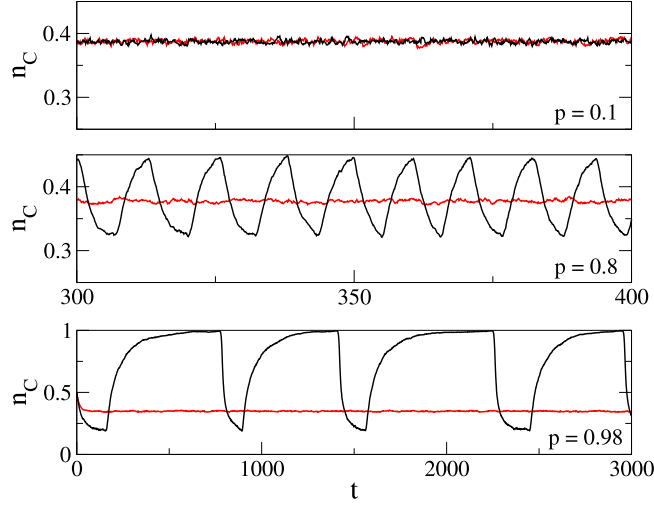$$f_\beta(i, j) = \left(1 + e^{-\beta[\phi(\sigma_i) - \phi(\sigma_j)]} \right)^{-1}. \tag{3}$$

Figure 1. Time series of the fraction of cooperators in an adaptive network for different rewiring rates p. When rewiring occurs almost at random ($\alpha = 0.1$, red), cooperation and defection coexist at a stationary level for all p. When players following the more successful strategy keep the link with high probability ($\alpha = 30$, black), oscillations appear as the rate of rewiring exceeds a critical threshold. Parameters: $N = 10^5$, $K = 10^6$, $\beta = 0.1$, $b = 1$, $c = 0.8$.

Otherwise, i.e., with probability $f_\beta(j, i) = 1 - f_\beta(i, j)$, the strategy of agent j is assigned to agent i. The parameter β is the selection intensity and corresponds to an inverse temperature. For small β the strategy adoption is almost random, whereas for $\beta \to \infty$ the more successful strategy is always adopted. Following established practice [8, 37, 42], we mainly focus on the case of weak selection (small β), which is known to be highly relevant for biology and supported by recent evidence for social systems [43]. Weak selection also implies that an agent's local neighbourhood is more important for an agent's strategy than the global information. While the global information (weakly) influences the probability that the agent's strategy changes in a given strategy update, the local neighbourhood governs the rates at which update events involving the focal agent occur.

In a similar fashion, we assume that the players using the more successful strategy are more likely to keep links during rewiring events. If a rewiring event occurs on a link connecting the agents i and j, the link is cut and then a new link is established between a randomly selected agent k and agent i (with probability $f_\alpha(i, j)$) or between k and j (with probability $f_\alpha(j, i)$). Here, we have used the Fermi function with selection intensity α to capture that agents following a successful strategy may find it easier to attract new contacts. Using the average payoff $\phi(\sigma)$ instead of the payoff obtained by individual agents prevents successful players from acquiring an unrealistically large number of links, which could otherwise lead to the formation of star-like topologies. It also implies that all agents using the same strategy are considered equivalent for the dynamics. We note that a similar assumption is made in the large class of models where the topological processes depend exclusively on the nodes' states instead of their fitness (e.g., [29, 30]).
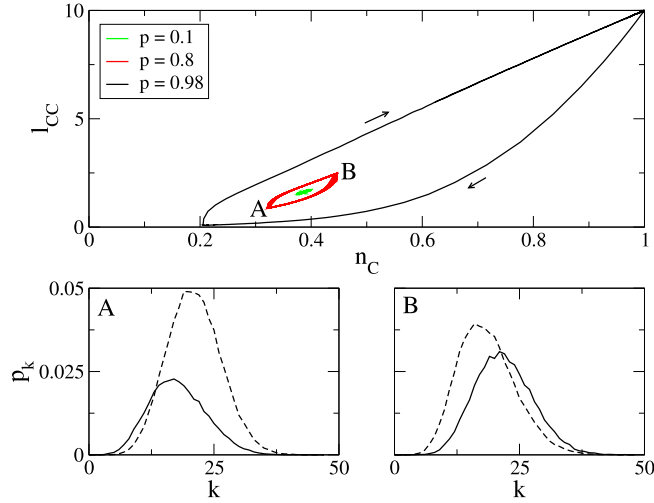
Figure 2. Cycles of cooperation and defection in agent-based simulations. Top: dynamics in the n_C-l_{CC}-plane for rewiring selection intensity $\alpha = 30$. Bottom: degree distributions of cooperators (——) and defectors (- - - -) for rewiring rate $p = 0.8$ at the two turning points of the cycle. Simulation parameters as in figure 1.

3. Simulation results

In order to explore the dynamics of the model, we run individual-based stochastic simulations for $N = 10^5$ and $K = 10^6$. Typical time series for different rewiring rates p are shown in figure 1. For weak selection ($\alpha, \beta \ll 1$), the system approaches a stable steady state where both strategies coexist. In this regime the stationary density of cooperators depends only weakly on p. If rewiring is strongly selective ($\alpha \gg \beta$), then the dynamics depend strongly on p. Stationary behaviour is still observed if p is small, but as p increases, the system undergoes a continuous transition in which the density of cooperators starts to oscillate. The same transition can also be observed for higher β, but is shifted to greater values of p. As p is increased further, the amplitude and period of the cycle grows. At higher p, long periods of almost full cooperation appear, which are interrupted by sudden episodes of defection.

To understand the onset of oscillations, note first that in the stationary state the average payoffs of cooperators and defectors have to be identical, so that strategy adoption and rewiring happen randomly. If, due to fluctuations, cooperators receive a slightly higher payoff than defectors, then the density of C-C links, l_{CC}, starts to increase due to the effect of the strongly selective rewiring, which tends to de-mix the network by accumulating links in the population with the higher payoff. The increasing l_{CC} constitutes a positive feedback increasing the payoff of the cooperators further. As agents adopt the cooperating strategy, the system approaches a state where both n_C and l_{CC} are high (point B in figure 2). In this state, strategy adoption can overcome the de-mixing effect of rewiring because adoption of the defecting strategy by cooperators creates many C-D links. The payoff of defectors rises rapidly, leading the system back to a mixed state, where a substantial number of agents are defecting.

4. Analytical results

To gain a deeper understanding we formulate a low-dimensional approximation of our model. The simplest possible model, a mean-field approximation of n_C, yields a one-dimensional ordinary differential equation (ODE), which cannot reproduce oscillatory long-term dynamics. We therefore use a moment-expansion approach [44], which describes the system on the level of the density of nodes and links. We treat n_C, l_{CC}, and l_{DD} as dynamical variables, whereas n_D and l_{CD} are given by the conservation laws $n_C + n_D = 1$ and $l_{CC} + l_{DD} + l_{CD} = \langle k \rangle /2$, where $\langle k \rangle = 2K/N$ denotes the mean degree.

The process that is most difficult to capture in the model is strategy adoption, because it does not only affect the focal link, but also all other links connecting to the agent whose strategy is changed. For instance, the average number of C-C links that are destroyed when a cooperator adopts the strategy of a defector depends on l_{DCC}, the number of D-C-C triplets per agent. To close the system we use the pair approximation $l_{XYZ} = (\eta_{XY}\eta_{YZ}/\eta_{XZ}) l_{XY} l_{YZ}/n_Y$, where $\eta_{AB} = 1 + \delta_{AB}$ denotes factors arising from symmetry. Using this procedure we obtain

$$\frac{\mathrm{d}}{\mathrm{dt}} n_C = q l_{CD} \left(f_\beta - \bar{f}_\beta \right), \tag{4}$$

$$\frac{\mathrm{d}}{\mathrm{dt}} l_{CC} = p n_C l_{CD} f_\alpha - p n_D l_{CC}$$
$$+ q l_{CD} \left(1 + \frac{l_{CD}}{n_D} \right) f_\beta - 2q \frac{l_{CC} l_{CD}}{n_C} \bar{f}_\beta, \tag{5}$$

$$\frac{\mathrm{d}}{\mathrm{dt}} l_{DD} = p n_D l_{CD} \bar{f}_\alpha - p n_C l_{DD}$$
$$+ q l_{CD} \left(1 + \frac{l_{CD}}{n_C} \right) \bar{f}_\beta - 2q \frac{l_{DD} l_{CD}}{n_D} f_\beta, \tag{6}$$

where we have introduced the abbreviated notation $f_\xi = \left(1 + e^{-\xi [\phi(C) - \phi(D)]} \right)^{-1}$ and $\bar{f}_\xi = 1 - f_\xi$. In (4), the first factor, $q l_{CD}$, denotes the rate of strategy adoption events while the second factor is the expected change in n_C in each such event. Analogously, the first two terms in (5) and (6) describe the gain and loss rates of the respective link density due to rewiring, while the third and fourth terms account for the link creation and loss due to strategy adoption.

In the regime of weak rewiring selection, $\alpha \ll 1$, and weak strategy selection, $\beta \ll 1$, our model reduces to previously studied systems in two important limiting cases. For fast rewiring ($p \approx 1$), which is almost random when α is small, the system evolves according to standard replicator dynamics in a well-mixed populations. On the other hand, when strategy adoption is much faster than rewiring, $p \ll 1$, the network is almost static and the modified replicator equation on graphs applies [45].

We now use the low-dimensional model, (4)–(6), to explore the system with the tools of nonlinear dynamics. A bifurcation diagram of the ODE system is in good agreement with the results of the agent-based simulations (figure 3). If rewiring is slow (small p), the system approaches an equilibrium in which cooperators and defectors coexist. For strongly selective rewiring ($\alpha \gg \beta$), however, a critical threshold p exists at which this steady state is destabilized in a supercritical Hopf bifurcation (point S in figure 3) and a stable limit cycle emerges, explaining the onset of oscillations.

As p is increased further, the amplitude of the limit cycle grows. Eventually, the cycle undergoes a homoclinic bifurcation as its upper turning point connects to the fully cooperative state ($n_C = 1$, $l_{CC} = \langle k \rangle /2$, $l_{DD} = 0$). Dynamically, this state is a saddle point, which the
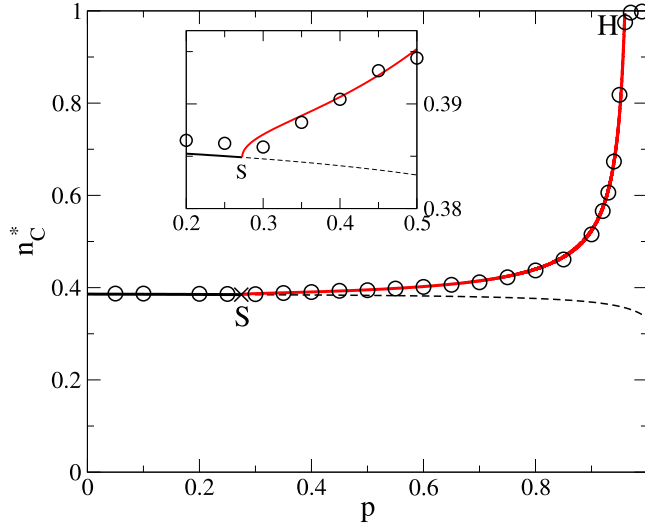
Figure 3. Bifurcation diagram for the case of strong rewiring selection ($\alpha = 30$). If rewiring is slow (small p), cooperators and defectors coexist in a stable steady state. At higher rewiring rates the stability is lost in a supercritical Hopf bifurcation (S), from which a stable limit cycle emerges. The limit cycle undergoes a homoclinic bifurcation in point H as it connects to a saddle point at $n_C = 1$. The lines show the stable (——) and unstable (- - - -) steady state, and the upper turning point of the limit cycle (red), computed in the low-dimensional model. Circles denote agent-based simulation results for $N = 10^5$, $K = 10^6$. The inset shows a blow-up of the bifurcation point S. See figure 1 for parameters.

cycle approaches along its stable manifold and leaves along the unstable manifold, forming a homoclinic loop. In the saddle point, the velocity at which the system moves along the cycle approaches zero. If observed at a random point in time, the ODE system is therefore found to be in the fully cooperative state with probability one.

Let us emphasize that the asymptotic full cooperation is a purely dynamical effect. The existence of a limit cycle shows that the time-averaged fitness of cooperators and defectors is equal. However, in the homoclinic bifurcation the time-average becomes meaningless as it has to be taken over infinite time, while for any finite time the cooperators dominate.

The deterministic description provided by the ODE system holds in the thermodynamic limit of large system size. In the agent-based model, full cooperation is an absorbing state of the strategy dynamics. Small systems can reach this state and remain at full cooperation. In larger systems ($N > 100$), the system slowly approaches the saddle point along the stable manifold, but is eventually carried over to the unstable manifold by fluctuations. Once on the unstable manifold, defection rapidly invades the population, launching the system into another round on the cycle before the fully cooperative state is approached again (figure 2). Because of their stochastic excitable nature, these invasions of defectors occur at irregular time intervals (figure 1 bottom), becoming longer with increasing system size.

In the two-parameter bifurcation diagrams shown in figure 4, the Hopf and homoclinic bifurcation points form lines, which separate parameter regions of qualitatively different long-term dynamics. The oscillatory dynamics and asymptotic full cooperation can be observed in a large parameter region, demonstrating the robustness of the observed phenomenon to the snowdrift game conditions (parametrized by the cost-to-benefit ratio $c/(2b - c)$ of mutual cooperation) and the rewiring selection strength α. For larger α, the Hopf bifurcation occurs
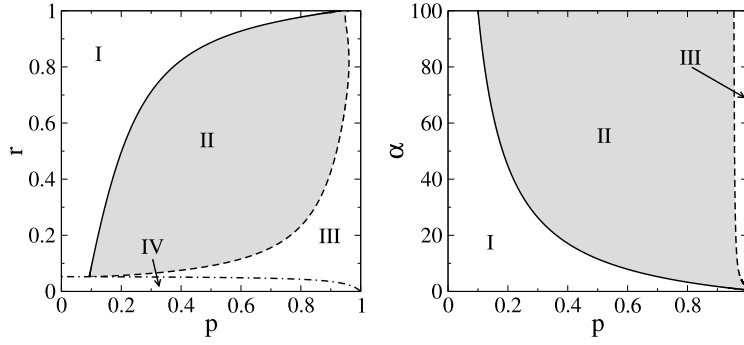
Figure 4. Two-parameter bifurcation diagrams showing the dependence on the rewiring rate p and the cost-to-benefit ratio [36] $r = c/(2b − c)$ (left, for $\alpha = 30$), and the rewiring selection strength α (right, for $b = 1$, $c = 0.8$), resp. In region I, cooperation and defection coexist in a stationary state. A Hopf bifurcation line (——) marks the transition to the oscillatory parameter region (II, shaded), which is bounded by a line of homoclinic bifurcations (- - - -) leading to asymptotic full cooperation (III). Stable full cooperation (IV) is reached via a transcritical bifurcation (— · —) if r is low. All bifurcation lines meet in a codimension-2 Takens-Bogdanov bifurcation. See figure 3 for additional parameters.

already at slower rewiring rates (figure 4, right), whereas the opposite is true for larger β (not shown): Although stronger replacement selection leads to oscillations at higher frequencies than before, it can be balanced by sufficiently fast rewiring and does not necessarily counteract the homoclinic mechanism.

We note that the homoclinic bifurcation line in figure 4 (right) approaches $p = 0.952 \approx 20/21$ for large α. On this line, 20 out of 21 network updates are rewiring events. Each rewiring event affects only a single link, while every strategy adoption event affects approximately $\langle k \rangle = 20$ links. Therefore, at $p = 20/21$ the links are in average affected at the rate $(1 − p)\langle k \rangle = p$ by strategy adoption and rewiring events, i.e., both processes operate on the same time scale.

Finally, we ask how our findings depend on our initial assumption that the agents have access to global, i.e., population-wide information in terms of $\phi(\sigma)$. To that end, we let the players estimate the fitness of strategies by averaging over a neighbourhood of nodes they can reach in a finite number of steps rather than over the whole population. Similar behaviour as before can be observed in simulations with such an "information horizon". In particular, fast selective rewiring leads to oscillations whenever this kind of neighbourhood information is used. As can be seen in figure 5, the amplitude and period of these oscillations grow with the information horizon (for fixed p). Therefore, access to the information from a sufficiently large neighbourhood is, in the present model, necessary to observe the homoclinic route to full cooperation.

5. Conclusions

In this paper, we proposed a model of cooperation on an adaptive network in which agents have access to non-local information. We observed that full cooperation is reached asymptotically through a global dynamical mechanism operating far from equilibrium. In addition to the results shown, we have verified that similar dynamics can be observed in other parameter regimes, such as stronger strategy selection, and in variants of the model.

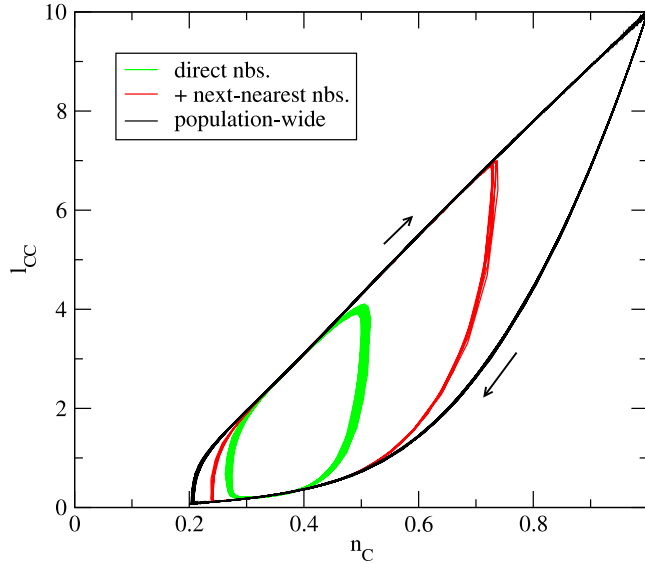In our model, asymptotic full cooperation is achieved dynamically, rather than by

Figure 5. Effect of increasing the agents' information horizon for fast rewiring in simulations. When the agents assess a strategy's fitness by averaging over the accumulated payoffs of the focal link's nearest neighbours only (green), a small limit cycle is obtained. It is larger when also the next-nearest neighbours are included (red). The size of the cycle grows with increasing averaging radius. The quasi-homoclinic cycle of long cooperative periods interrupted by short bursts of defection is recovered when the average is taken over the whole population (black). Parameters as in figure 1, $p = 0.98$.

assembling characteristic topologies that allow cooperators to thrive. In finite populations, spontaneous collapses of highly cooperative states are observed. The homoclinic mechanism in combination with noise may thus be a relevant ingredient in the systemic failure of cooperation that is found in real world systems.

Although it is conceivable that global information can be accessible in social systems through, e.g., general beliefs, rumours, or the mass media, we note that this assumption is not necessary for the oscillatory behaviour. When the agents are restricted to a finite "information horizon", the present model still exhibits oscillations.

A sufficiently large information horizon is necessary to observe the homoclinic transition. We note, however, that in a different model in which local update rules are assumed and which explicitly enforces realistic limits on the node degrees, a similar transition was found numerically [17]. We believe that in the present model, the homoclinic mechanism only requires large information horizons because they prevent the formation of unrealistic star-like topologies, which would otherwise stabilize the dynamics.

More work is certainly necessary to explore the role of the homoclinic mechanism in nature. This work will, however, offer the intriguing possibility to identify and understand the dynamical features promoting cooperation in many systems, and may reveal what causes the sudden collapses of cooperative behaviour observed in the human population.

Acknowledgments

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